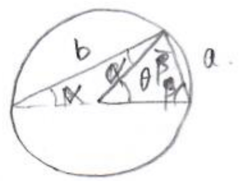
Date: 2023 Mar 18 and 2022 Apr 21

Trigonometry

1



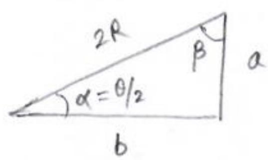
chord $(\theta) = a$

chord $(180-\theta) = b$

$\alpha + \beta = 90$ and $\alpha = \theta/2$.

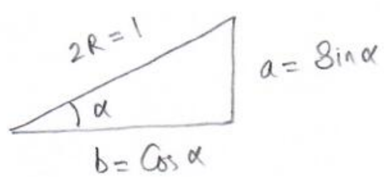
→ So, the sides of a right triangle are chords in a circle.

2



Thus, trigonometry is the study of chords in terms of angles. Or, the relation between angles and sides in a right triangle.

3 Definition



Immediately we have

$$\sin^2\alpha + \cos^2\alpha = 1.$$

4 Question: 1 Find side $a = \sin\alpha$ as a function of angle α.

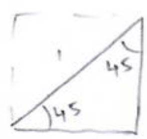
2 Find side b as a funch of angle α.

5. $\underline{\underline{90^\circ}}$

$$\sin 90^\circ = 1$$

$$\cos 90^\circ = 0$$

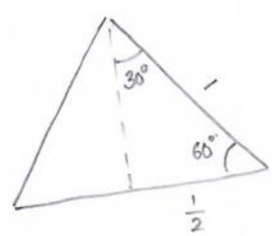
6. $\underline{45^\circ}$ (Square)



$$2 \sin^2 45 = 1 \qquad \sin 45 = \frac{1}{\sqrt{2}}$$

$$2 \cos^2 45 = 1 \qquad \cos 45 = \frac{1}{\sqrt{2}}$$

7. $\underline{\underline{30^\circ \text{ and } 60^\circ}}$ (Equilateral triangle)

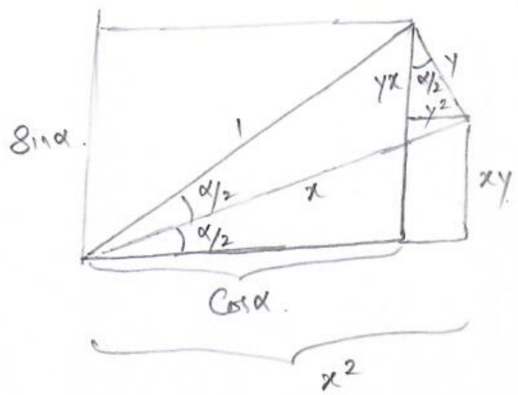


$$\sin 30 = \frac{1}{2} \qquad \cos 30 = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$$

$$\cos 60 = \frac{1}{2}$$

8. The above are isolated examples. Can we think of a strategy to find $\sin\alpha$ and $\cos\alpha$ of any angle? Addition formula provides such an option.

(9) Consider.



Find:

$x = \cos\frac{\alpha}{2}$

$y = \sin\frac{\alpha}{2}$

(10)

$$\begin{aligned}\cos\alpha &= x^2 - y^2 \\ &= x^2 - (1 - x^2) \\ &= 2x^2 - 1\end{aligned}$$

$$x = \cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}$$

(11)

$$\sin\alpha = xy + xy$$

$$\begin{aligned}y &= \frac{\sin\alpha}{2x} \\ &= \frac{\sqrt{1-\cos^2\alpha}}{2\sqrt{\frac{1+\cos\alpha}{2}}} \\ &= \sqrt{\frac{1-\cos\alpha}{2}}\end{aligned}$$

$$y = \sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}$$

(12) Example: $\alpha = 30$

$$\cos 15 = \sqrt{\frac{1+\cos 30}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\sin 15 = \sqrt{\frac{1-\cos 30}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

(13) Example: $\alpha = 45$

$$\cos 45 = \sqrt{\frac{1+\cos 45}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

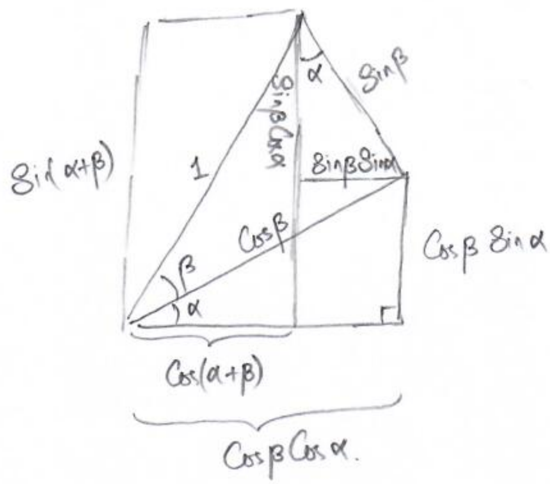
$$\sin 45 = \sqrt{\frac{1-\cos 45}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

(14) Using these ideas

140 BC : Hipparchus
evaluated and constructed the first trigonometric table (in multiples of 7.5°)

200 AD : Ptolemy
prepared the 'Table of Chords' (in multiples of 0.5°) which remained the state of art until 1500s.

(15) Addition formula.



$$(90-\beta) - (90-\alpha-\beta) = \alpha$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$