

Worksheet: Retarded Time

Potpourri Seminar, Southern Illinois University–Carbondale

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The path of a relativistic particle 1 moving along a straight line with constant (proper) acceleration g is described by the equation of a hyperbola

$$z_1(t) = \sqrt{c^2t^2 + z_0^2}, \quad z_0 = \frac{c^2}{g}. \quad (1)$$

This is the motion of a particle that comes to existence at $z_1 = +\infty$ at $t = -\infty$, then ‘falls’ with constant (proper) acceleration g . If we choose $x_1(0) = 0$ and $y_1(0) = 0$, the particle ‘falls’ keeping itself on the z -axis, comes to stop at $z = z_0$, and then returns back to infinity. Consider another relativistic particle 2 undergoing hyperbolic motion given by

$$z_2(t) = -\sqrt{c^2t^2 + z_0^2}, \quad z_0 = \frac{c^2}{g}. \quad (2)$$

This is the motion of a particle that comes to existence at $z_2 = -\infty$ at $t = -\infty$, then ‘falls’ with constant (proper) acceleration g . If we choose $x_2(0) = 0$ and $y_2(0) = 0$, the particle ‘falls’ keeping itself on the z -axis, comes to stop at $z = -z_0$, and then returns back to negative infinity. The world-line of particle 1 is the blue curve in Figure , and the world-line of particle 2 is the red curve in Figure . Using geometric (diagrammatic) arguments might be easiest to answer the following. Imagine the particles are sources of light (imagine a flash light pointing towards origin).

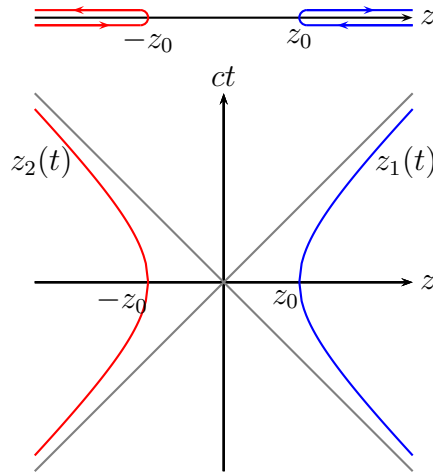


Figure 1: World-line of two particles moving with constant proper acceleration.

1. At what time will the light from particle 1 first reach particle 2? Where are the particles when this happens?
2. At what time will the light from particle 2 first reach particle 1? Where are the particles when this happens?
3. Can the particles communicate with each other?
4. Can the particles ever detect the presence of the other? In other words, can one particle be aware of the existence of the other? What can you deduce about the observable part of our universe from this analysis?