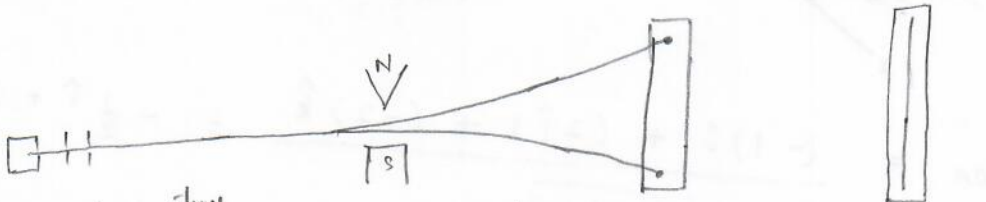
Date: 2022 Aug 20

Title: Quantum mechanics: Measurement disturbs the system

Event: Lecture Series in Theoretical Physics

Venue: Sphics Science Center

① Stern - Gerlach experiment (1922)



beam of silver atoms

magnetic moment

$\vec{\mu} = \frac{q}{2m}\vec{L}$

$\vec{L} = \frac{\hbar}{2}\vec{\sigma}$

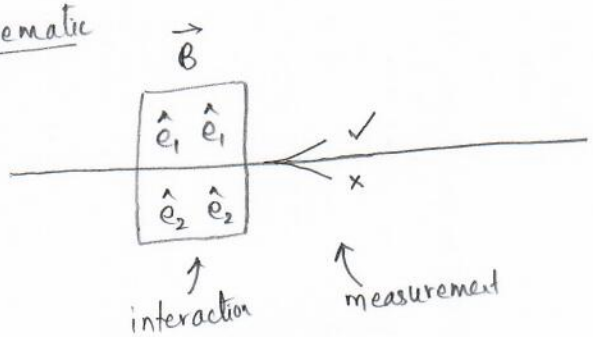
magnetic field interaction

$H = -\vec{\mu}\cdot\vec{B}$

photographic plate

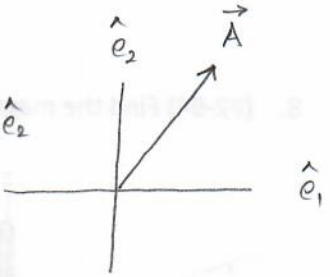
expected result

② Schematic



Let us consider this two-state setup in series.

③ Vector $\vec{A}$



→ basis dependent description: $\vec{A} = A_1 \hat{e}_1 + A_2 \hat{e}_2$

→ completeness relation: $\overleftrightarrow{1} = \hat{e}_1 \hat{e}_1 + \hat{e}_2 \hat{e}_2$

→ orthogonality relation: $\hat{e}_1 \cdot \hat{e}_1 = 1 \quad \hat{e}_1 \cdot \hat{e}_2 = 0$

$\hat{e}_2 \cdot \hat{e}_1 = 0 \quad \hat{e}_2 \cdot \hat{e}_2 = 1$

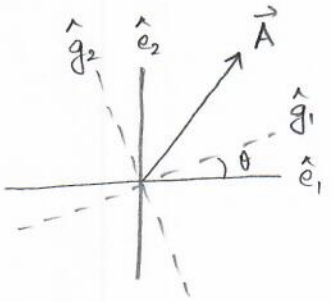
→ projection operator: $(\hat{e}_1 \hat{e}_1) \cdot \vec{A} = \hat{e}_1 \, \hat{e}_1 \cdot \vec{A} = \hat{e}_1 A_1$

$(\hat{e}_2 \hat{e}_2) \cdot \vec{A} = \hat{e}_2 \, \hat{e}_2 \cdot \vec{A} = \hat{e}_2 A_2$

$$\left[(\hat{e}_1 \hat{e}_1) + (\hat{e}_2 \hat{e}_2)\right] \cdot \vec{A} = \vec{A}$$

④ $\hat{e}_1 \hat{e}_1 + \hat{e}_2 \hat{e}_2 = \overleftrightarrow{1}$

$\hat{g}_1 \hat{g}_1 + \hat{g}_2 \hat{g}_2 = \overleftrightarrow{1}$



⑤ dissociated atoms $\dfrac{\hat{e}_1 \hat{e}_1 \ \checkmark}{\hat{e}_2 \hat{e}_2 \ \times} \ \hat{e}_1$

$\vec{A} \cdot (\hat{e}_1 \hat{e}_1) = A_1 \hat{e}_1 \xrightarrow{\text{normalize}} \hat{e}_1$

⑥ $\hat{e}_1 \ \dfrac{\hat{e}_1 \hat{e}_1 \ \checkmark}{\hat{e}_2 \hat{e}_2 \ \times} \ \hat{e}_1$

$A = \hat{e}_1 \cdot (\hat{e}_1 \hat{e}_1) \cdot \hat{e}_1 = 1$

$P = |A|^2 = 1$

source — interaction — selection measurement — observer

(7) interaction with measurement

$\hat{e}_1$ — $\hat{e}_1\hat{e}_1$ ✓ / $\hat{e}_2\hat{e}_2$ × — $\hat{e}_1$: $A = \hat{e}_1 \cdot (\hat{e}_1\hat{e}_1) \cdot \hat{e}_1 = 1$, $P = 1$

$\hat{e}_1$ — $\hat{e}_1\hat{e}_1$ ✓ / $\hat{e}_2\hat{e}_2$ × — $\hat{e}_2$: $A = \hat{e}_1 \cdot (\hat{e}_1\hat{e}_1) \cdot \hat{e}_2 = 0$, $P = 0$

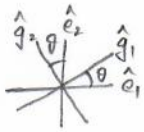
$\hat{e}_1$ — $\hat{e}_1\hat{e}_1$ × / $\hat{e}_2\hat{e}_2$ ✓ — $\hat{e}_1$: $A = \hat{e}_1 \cdot (\hat{e}_2\hat{e}_2) \cdot \hat{e}_1 = 0$, $P = 0$

$\hat{e}_1$ — $\hat{e}_1\hat{e}_1$ × / $\hat{e}_2\hat{e}_2$ ✓ — $\hat{e}_2$: $A = \hat{e}_1 \cdot (\hat{e}_2\hat{e}_2) \cdot \hat{e}_2 = 0$, $P = 0$

interaction without measurement

$\hat{e}_1$ — $\hat{e}_1\hat{e}_1$ ✓ / $\hat{e}_2\hat{e}_2$ ✓ — $\hat{e}_1$: $A = \hat{e}_1 \cdot (\hat{e}_1\hat{e}_1 + \hat{e}_2\hat{e}_2) \cdot \hat{e}_1 = 1$, $P = 1$

$\hat{e}_1$ — $\hat{e}_1\hat{e}_1$ ✓ / $\hat{e}_2\hat{e}_2$ ✓ — $\hat{e}_2$: $A = \hat{e}_1 \cdot (\hat{e}_1\hat{e}_1 + \hat{e}_2\hat{e}_2) \cdot \hat{e}_2 = 0$, $P = 0$



(8) interaction with measurement

$\hat{e}_1$ — $\hat{g}_1\hat{g}_1$ ✓ / $\hat{g}_2\hat{g}_2$ × — $\hat{e}_1$: $A = \hat{e}_1 \cdot (\hat{g}_1\hat{g}_1) \cdot \hat{e}_1 = \cos^2\theta$, $P = \cos^4\theta$

$\hat{e}_1$ — $\hat{g}_1\hat{g}_1$ ✓ / $\hat{g}_2\hat{g}_2$ × — $\hat{e}_2$: $A = \hat{e}_1 \cdot (\hat{g}_1\hat{g}_1) \cdot \hat{e}_2 = \cos\theta \sin\theta$, $P = \cos^2\theta \sin^2\theta$

$\hat{e}_1$ — $\hat{g}_1\hat{g}_1$ × / $\hat{g}_2\hat{g}_2$ ✓ — $\hat{e}_1$: $A = \hat{e}_1 \cdot (\hat{g}_2\hat{g}_2) \cdot \hat{e}_1 = \sin^2\theta$, $P = \sin^4\theta$

$\hat{e}_1$ — $\hat{g}_1\hat{g}_1$ × / $\hat{g}_2\hat{g}_2$ ✓ — $\hat{e}_2$: $A = \hat{e}_1 \cdot (\hat{g}_2\hat{g}_2) \cdot \hat{e}_2 = -\sin\theta\cos\theta$, $P = \cos^2\theta \sin^2\theta$

interaction without measurement

$\hat{e}_1$ — $\hat{g}_1\hat{g}_1$ ✓ / $\hat{g}_2\hat{g}_2$ ✓ — $\hat{e}_1$: $A = \hat{e}_1 \cdot \overleftrightarrow{1} \cdot \hat{e}_1 = 1$, $P = 1$

$\hat{e}_1$ — $\hat{g}_1\hat{g}_1$ ✓ / $\hat{g}_2\hat{g}_2$ ✓ — $\hat{e}_2$: $A = \hat{e}_1 \cdot \overleftrightarrow{1} \cdot \hat{e}_2 = 0$, $P = 0$

⑨ Summary

interaction with measurement

$$\hat{e}_1 \;\; \frac{\hat{g}_1\hat{g}_1 \;\checkmark}{\hat{g}_2\hat{g}_2 \;\times} \;\; \hat{e}_1$$

$$A = \hat{e}_1 \cdot (\hat{g}_1\hat{g}_1) \cdot \hat{e}_1$$

$$= \cos^2\theta$$

$$P = \cos^4\theta \xrightarrow{\theta = 45^\circ} \frac{1}{4}$$

interaction without measurement

$$\hat{e}_1 \;\; \frac{\hat{g}_1\hat{g}_1 \;\checkmark}{\hat{g}_2\hat{g}_2 \;\checkmark} \;\; \hat{e}_1$$

$$A = \hat{e}_1 \cdot \overleftrightarrow{1} \cdot \hat{e}_1$$

$$= 1$$

$$P = 1$$

Measurement disturbs the system!

⑩ David Bohm (chapter 22, page 584)

Observation requires an interaction, however, an interaction does not establish an observation.