

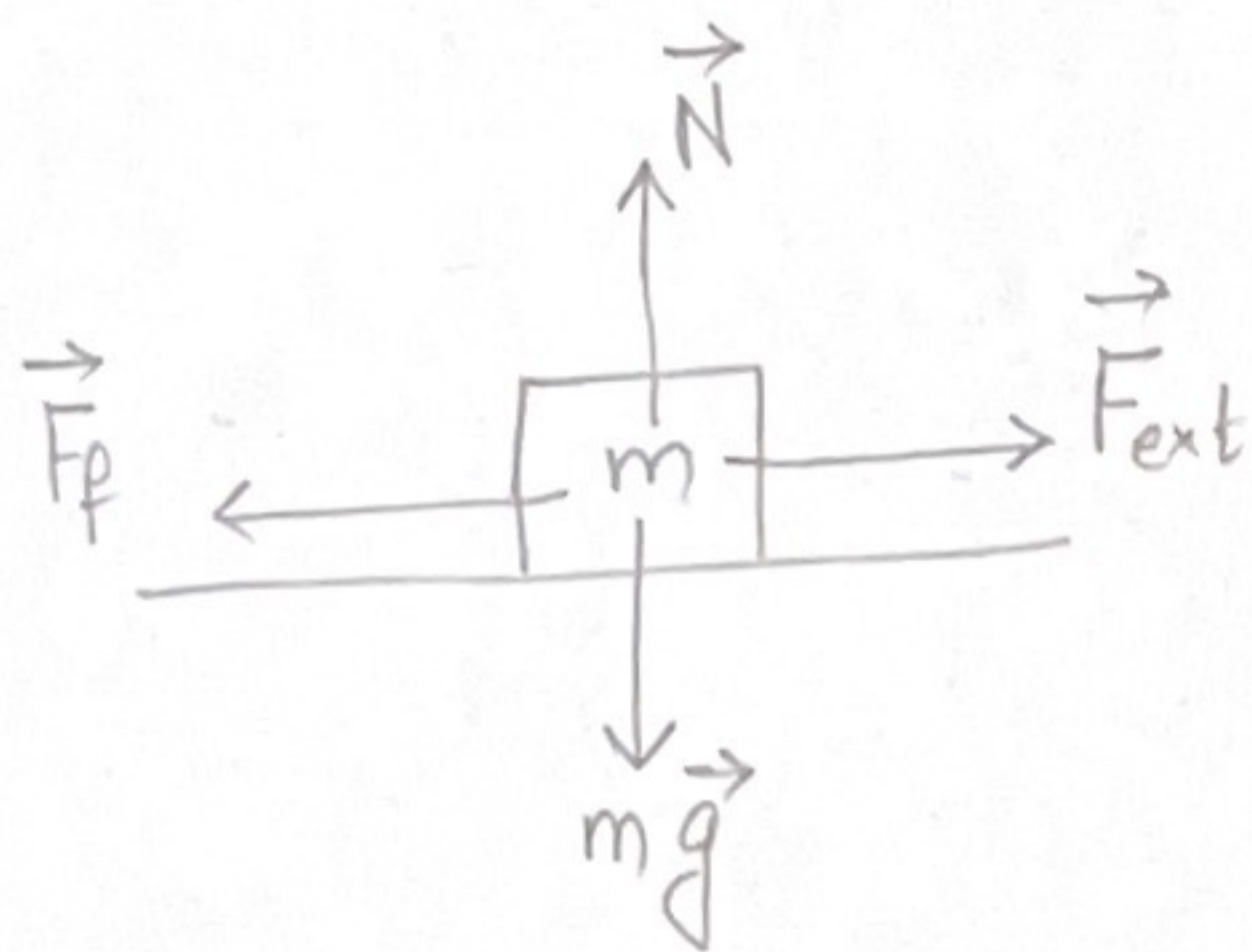
Date: 2022 June 25

Title: Direction of friction on rolling wheels.

Event: Lecture-Series in Theoretical Physics

Venue: Sphics Science Center.

① Newton's laws of motion



$$m\vec{a} = m\vec{g} + \vec{N} + \vec{F}_{ext} + \vec{F}_f$$

$$ma = F_{ext} - F_f$$

$$0 = N - mg$$

② Amontons-Coulomb model of friction

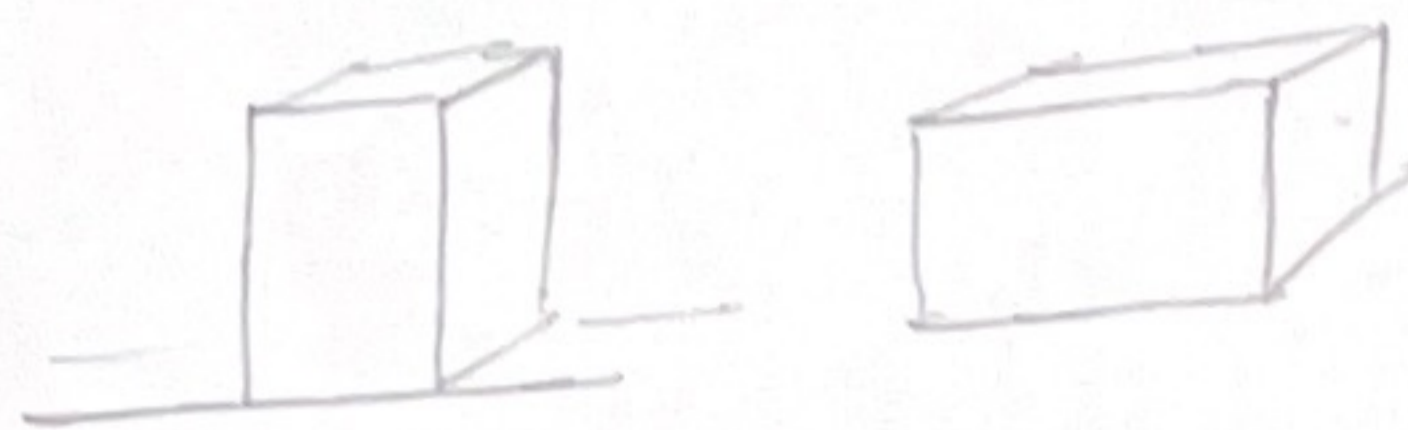
$$F_f \begin{cases} \leq \mu_s N \\ = \mu_k N \end{cases}$$

→ F_f is independent of area.

→ F_f proportional to N .

→ μ_s depends on the molecular forces at the interface.

Area:



dependence of N :

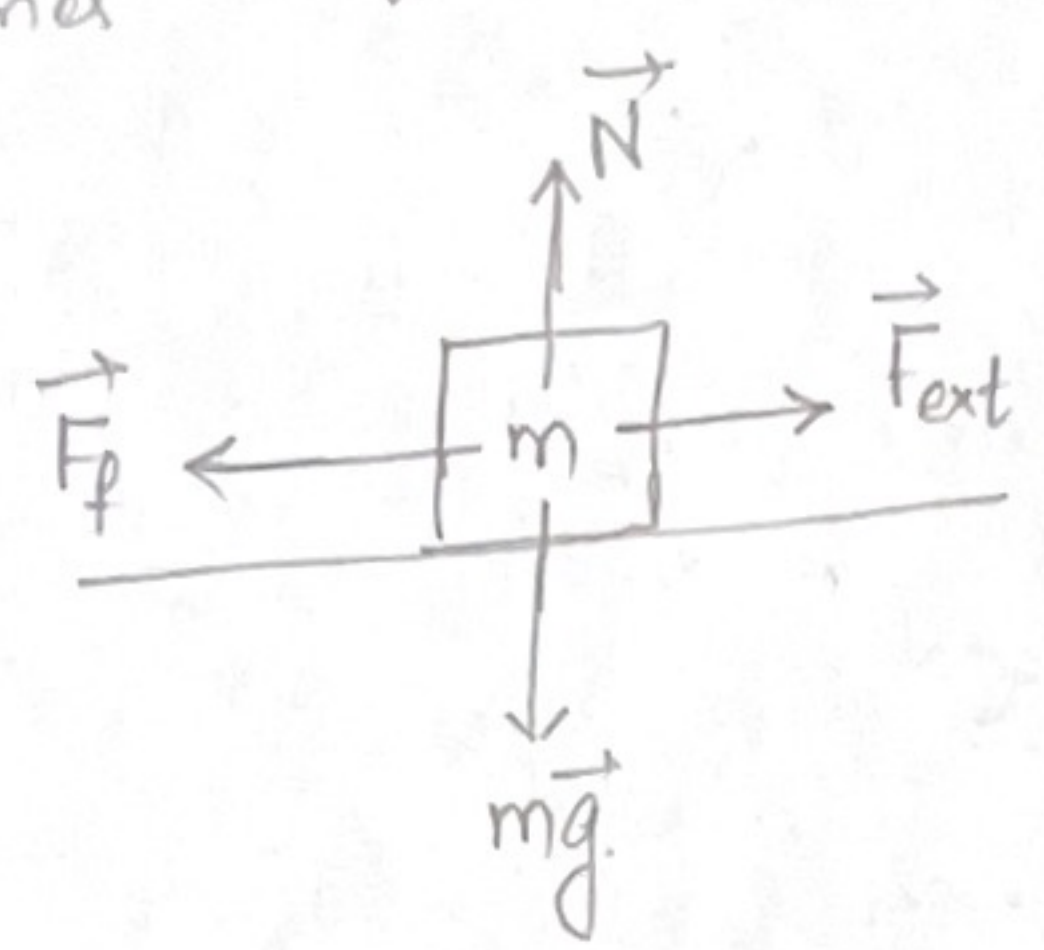


→ 'real' contact area is proportional to N .

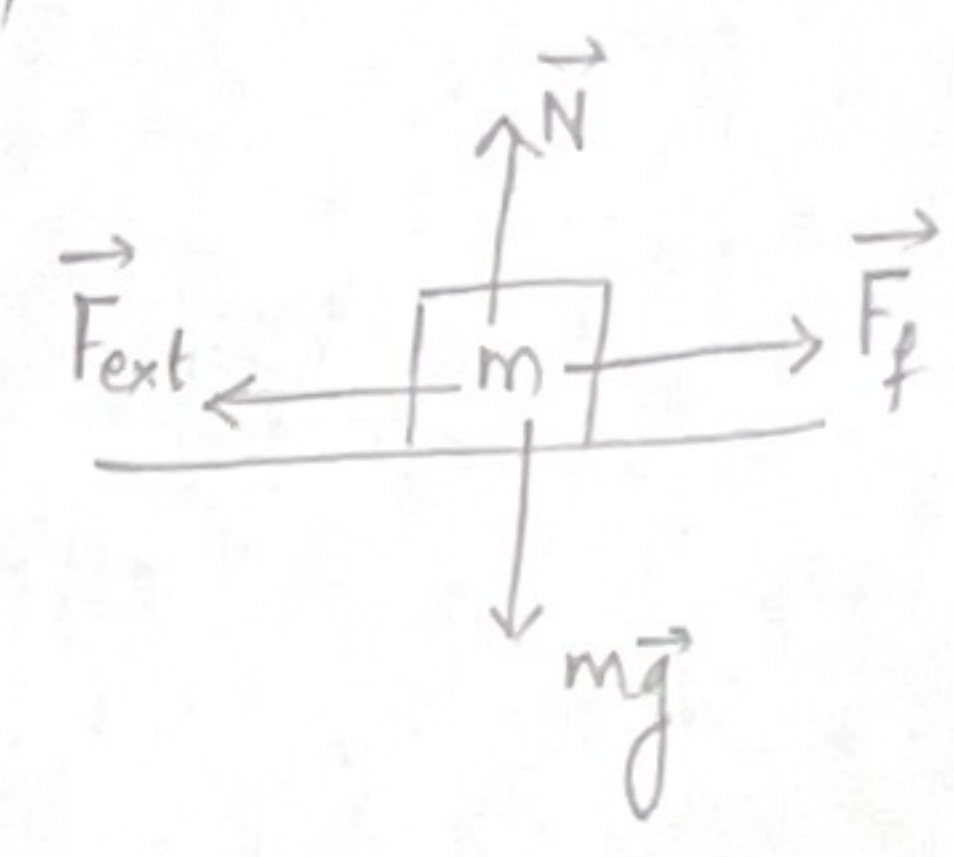
→ Hertz's model of elasticity.

③ Friction is a response to an external force that tries to move the body.

$a=0$



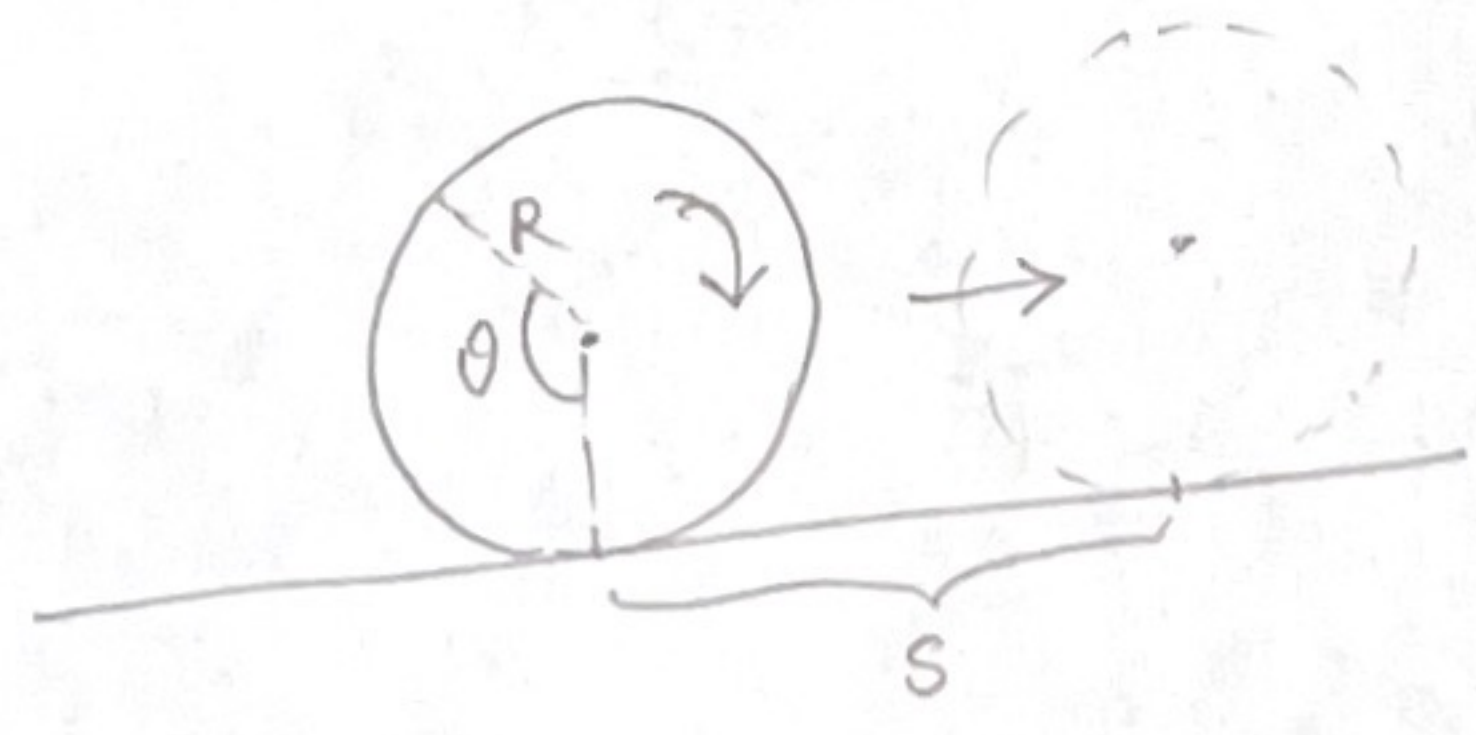
(OR)



Observe that for $a=0$ $F_{ext} = F_f$

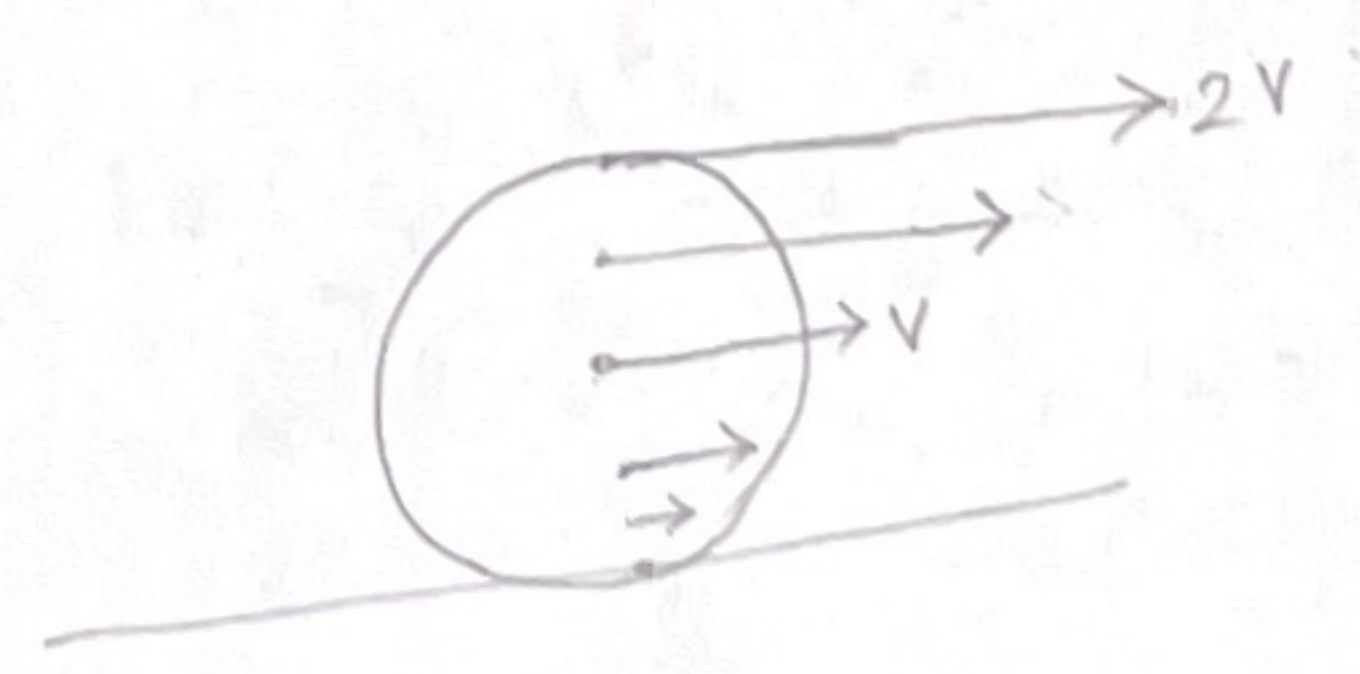
④ Perfectly rolling wheel

straight-line motion + rolling motion.



- $s < \theta R$ (slipping, snow)
- $s = \theta R$ (perfect rolling)
- $s > \theta R$ (sliding, ice)

- $v < \omega R$
- $v = \omega R$
- $v > \omega R$



during perfect rolling the point making contact with the surface is (relatively) at rest.

⑤ Engine assisted rolling



linear motion: $ma = F_f$ (assists linear motion)

rolling motion: $I\alpha = \tau_{ext} - F_f R$ (resists rolling motion)

⑥ If we presume (incorrectly) that friction is in the backward direction we will have.

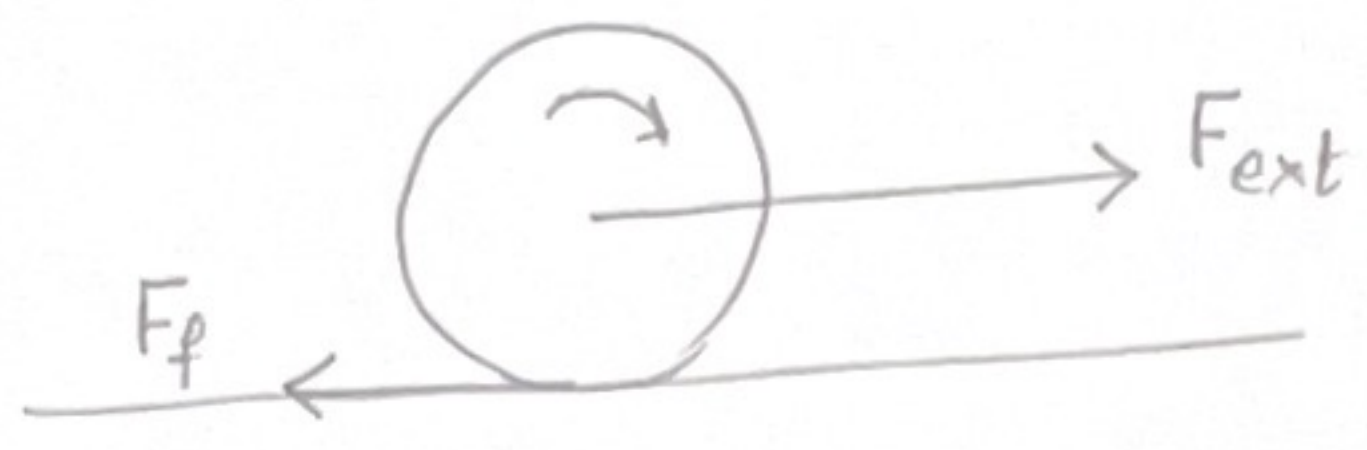
linear motion: $ma = -F_f$

rolling motion: $I\alpha = \tau_{ext} + F_f R$



For initial velocity $v_i = \omega_i R$, v_i tends to decrease but ω_i tends to increase. So, the condition for perfect rolling, $v = \omega R$, can not be sustained in this case.

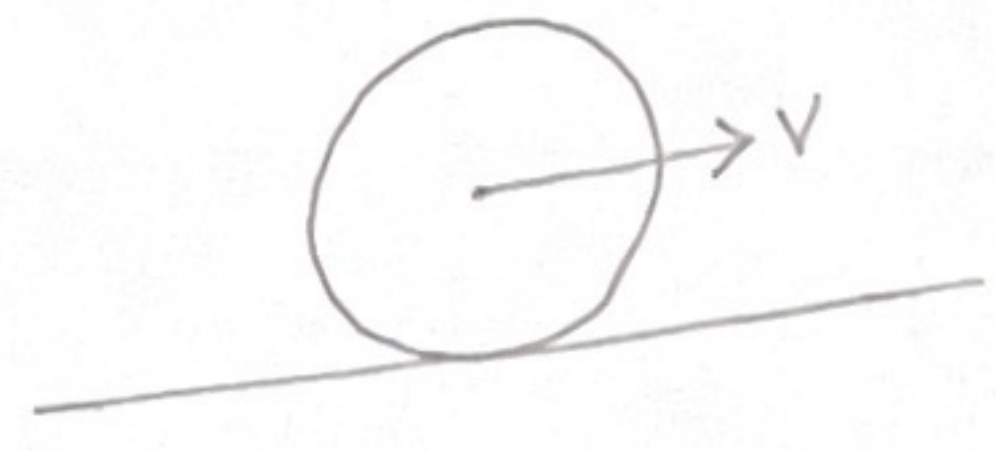
⑦ Rolling by pulling on the axis



linear motion: $ma = F_{ext} - F_f$ (resists linear motion)

rolling motion: $I\alpha = F_f R$ (assists rolling motion)

⑧ Perfect rolling in the absence of external forces/torques.



no force of friction

If force of friction is present, say in the forward direction it will assist (resist) linear motion (rolling motion) and thus immediately not be able to sustain the condition of perfect rolling $v = \omega R$. This is because v will increase and ω will decrease.